Josh Bowden

Professor Sasaki

CS 330 – Discrete Structures

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Homework 5

1. Points = 2: There are 4 major auto routes from Boston to Detroit and 6 from Detroit to Los Angeles. How many major auto routes are there from Boston to Los Angeles via Detroit?
2. Points = 3: How many positive integers between 5 and 31
   1. Are divisible by 3? List them.
   2. Are divisible by 4? List them.
   3. Are divisible by 3 and by 4? List them.
3. Points = 3: How many strings of 4 decimal digits
   1. Do not contain the same digit twice?

4536

* 1. End with an even digit?

4500

* 1. Have exactly 3 digits that are 9s?

35

1. Points = 6: In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if
   1. The bride must be in the picture?

90,720

* 1. Both the bride and groom must be in the picture?

50,400

* 1. Exactly one of the bride and the groom is in the picture?

40,320

1. Points = 2: A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random (without replacement) without looking at them.
   1. How many balls must she select to be sure of having at least 3 balls of the same color?

5

* 1. How many balls must she select to be sure of having at least 3 blue balls?

13

1. Points = 2: Suppose that there are 9 students in a discrete mathematics class at a small college.
   1. Show that the class must have at least 5 male students or at least 5 female students.  
      By the Pigeonhole Principle, there must be at least 5 male students or at least 5 female students.

In the case that there are at least 5 male students, there must be at least 4 female students so that there are 9 students in the class.

In the case that there are at least 5 female students, there must be at least 4 female students so that there are 9 students in the class.

* 1. Show that the class must have at least 3 male students or at least 7 female students.

By the Pigeonhole Principle, there must be at least 5 male students or at least 5 female students.

In the case that there are at least 3 male students, there must be at least 6 female students so that there are 9 students in the class.

In the case that there are at least 7 female students, there must be at least 2 female students so that there are 9 students in the class.

1. Points = 2: A computer network has 6 computers with computer directly connected to at least 1 of the others. Show that there are at least 2 computers in the network that are directly connected to the same number of other computers.

By the generalized pigeonhole principle, since one of the 6 computers (n) has only 5 other computers to be directly connected to (k), then . Therefore, there must be at least 2 computers that are connected to the same number of other computers.

1. Points = 2: Let S = {1, 2, 3, 4, 5}.
   1. How many 3-combinations of S are there? List them.
   2. How many 3-permutations of S are there? Show how to use your answer from part (a) to get all the permutations (you don’t have to actually list all of them individually).

Since and , then .

1. Points = 2: There are 6 different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

720

1. Points = 4: How many bit strings of length 12 contain
   1. Exactly three 1s?

220

* 1. At most three 1s?

299

* 1. At least three 1s?

4,017

* 1. An equal number of 0s and 1s?

924

1. Points = 4: A coin is flipped 8 times where each flip comes up either heads or tails. How many possible outcomes
   1. Are there in total?

256

* 1. Contain exactly 3 heads?

56

* 1. Contain at least 3 heads?

219

* 1. Contain the same number of heads and tails?

70

1. Points = 4: How many ways are there for 10 women and 6 men to stand in a line so that no 2 men stand next to each other?

Of the 13 different formations based on gender, there are 250,952,593,541 different ways to position each person.

1. Points = 4: Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with 6 members if it must have more women than men?

Of the 3 different formations based on gender ignoring order, there are 96,460 different way to form a committee with different people.

**Reasoning:**

Since, the 3 formations ignoring order are:

such that for each arrangement, there are different ways to form a committee with different people for each gender (where and is the total number of women and men respectively, and and is the number of women and men respectively chosen for a given formation). The number of combinations for each formation is then summed together since the formations follow the sum rule since only one formation can be chosen at a time.